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Volume 74 Number 5 2025

Spirito, D.:

The complete integral closure of a Prüfer domain is a topological property 137

Bacelar, L., Llibre, J.:

Reversible nilpotent centers with cubic nonlinearities 138

Stanciu, M.:

Best proximity for Istrătescu mappings on generalized metric spaces 139

Das, M., Nahak, C., Mohapatra, R.N.:

Existence and Arcwise connectedness of Robust solutions for uncertain generalized multi-objective optimization problems 140

García, G.:

A generalization of Browder's Theorem to parametric DND-condensing mappings 141

Öz, N.M.P., Türkmen, B.N.:

Zariski subhyperspace topology on hyperideals 142

Mohammadzadeh Karizaki, M., Sohrabi, M.:

Applications of polar decomposition of EP operators 143

Durdiev, D.K., Suyarov, T.R.:

Inverse coefficient problem for the fractional wave equation with non-local initial-boundary and integral type overdetermination conditions 144

Abdelwahab, H., Kaygorodov, I., Sartayev, B.:

Shift associative algebras 145

Jabbarzadeh, M.R., Sohrabi, M.:

Generalized inverse and geometric mean of Lambert conditional operators 146

Monaim, H., Faress, M.:

Uncertainty principles for Fourier transform on the Cayley Heisenberg group 147

Hien, N.T.T.:

Some results on the concept of stochastic domination with applications to weak laws of large numbers for double arrays of random variables 148

Jornet, M., Nieto, J.J.:

Representation of continuous linear functionals in terms of probabilistic fractional differential operators 149

Najati, A., Tareeghee, M.A.:

Hyers–Ulam stability and hyperstability of a cubic functional equation of Euler–Lagrange type on restricted domains 150

He, Y., Jin, Y.:

Universal inequalities for eigenvalues of the buckling problem of arbitrary order on domains in an Euclidean space 151

Franco, D., Sarti, A.:	
Intersection cohomology and Severi varieties of quartic surfaces.....	152
Bouchama, C., Aid, O.F., Senoussaoui, A.:	
On the Hörmander classes of bilinear Fourier integral operators.....	153
Wang, Y., Liu, Q., Chen, L., Tan, X., Sarfraz, M.:	
On the $L_{\gamma,j}(\xi, \eta, X)$ constant for the Banaś-Frączek space	154
González Rodríguez, R., Rodríguez Raposo, A.B.:	
Categorical equivalences for Hopf trusses and their modules	155
Fuentes, E., Garza, L.E., Velásquez, F.:	
On multiplicative Jacobi polynomials and function approximation through multiplicative series	156
Belguith, M., Moalla, N., Walha, I.:	
Spectral analysis of one sided operator matrix and application to a radiative transfer equations in a channel.....	157
Ammar, A., Mnif, M., Souhail, A.:	
Discrete spectrum and polaroid linear relations	158
George, S., Gopal, M., Muniyasamy, M.:	
Improved convergence analysis for an at least fourth and at least sixth order parametric family of iterative methods for nonlinear system	159
Unni, A.S., Pragadeeswarar, V.:	
Existence and uniqueness of fractals in fuzzy metric space via best proximity point	160
Badea, G., Cilia, R., Gutiérrez, J.M.:	
Spaces of integral and nuclear multilinear mappings	161
Mamanazarov, A., Khalilov, K., Kodiraliev, A.:	
Direct and inverse source problems with non-local boundary conditions for a time-fractional time and space degenerate heat equation	162
Prasad, C.:	
Correction: Semilocal convergence of the higher order method in riemannian manifolds	163
Bouaam, H., El Ouarabi, M., Melliani, S.:	
Kirchhoff-type double-phase problems with variable exponents and logarithmic nonlinearity in Musielak-Orlicz Sobolev spaces	164
Sadouk, E.M., Benchiheb, O., Amouch, M.:	
Disjoint topologically recurrent operators	165
Jeridi, N.:	
Some results on higher order (A, m) -symmetric commuting tuples of operators on a Hilbert space	166
Amoroso, E., D'Agui, G., Vergallo, P.:	
Periodic solutions for a second order system of neural network type	167
Ha, N.S.:	
Inertial projection-type algorithms for systems of split equality problems in Hilbert spaces	168

Christ, K.:

Special divisors in special divisor classes on graphs 169

Karpenko, N.A.:

Poincaré duality for upper motives of absolutely simple algebraic groups 170

Chems-Eddin, M.M., El Mamry, H.:

On the unit group of some multiquadratic fields 171

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Abstract

We show that the prime spectrum of the complete integral closure D^* of a Prüfer domain D is completely determined by the Zariski topology on the spectrum $\text{Spec}(D)$ of D .

Keywords Complete integral closure · Prüfer domains · Zariski topology

Mathematics Subject Classification 13F05 · 13B22

1 Introduction

Let D be an integral domain with quotient field K . An element $z \in K$ is *almost integral* over D if there is a nonzero element $c \in D$ such that $cz^n \in D$ for every positive integer n . The set of all the elements of K that are almost integral over D is a ring [5, Theorem 13.1], called the *complete integral closure* of D , and D is said to be *completely integrally closed* if $D = D^*$. However, the term "closure" is a misnomer, as D^* need not to be completely integrally closed: the first example of this phenomenon was given by Gilmer and Heinzer [6, Example 1], and it was shown later that this can happen also for Prüfer domains [7], even in the two-dimensional case [11]. It is also possible that the process of taking repeatedly the complete integral closure never stabilizes [9].

The purpose of this paper is to show that, for a Prüfer domain D , the complete integral closure D^* (or rather, the spectrum of D^*) is uniquely determined by the spectrum $\text{Spec}(D)$ of D and by its Zariski topology. We do this by first reducing to the case of Bézout domains (by using the use of the Kronecker function ring) and then by characterizing the condition $D = D^*$ through the density of a particular enlargement of the closed cocompact sets containing P (Theorem 2.9). As a consequence, we prove that, for a Prüfer domain, the condition of being completely integrally closed depends uniquely on $\text{Spec}(D)$, and we prove the necessary condition under which D^* is completely integrally closed (Proposition 2.15). We conclude the complete integral closure of D with the sets of minimal primes of the finitely

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